Current Research Journal of Social Sciences 2(4): 255-261, 2010
ISSN: 2041-3246

Submitted Date: June 25, 2010 Accepted Date: August 19, 2010 Published Date: September 06, 2010

A Mathematical Model of Democratic Elections

Mato Nagel
Center for Nephrology and Metabolic Disorders, A.-Schweitzer-Ring 32,
D-02943 Weisswasser, Germany

Abstract: Democratic election is the preferred method for determining political administrators nowadays. The intention is to find the best possible leader in order to improve the group's competitiveness and success. Though preferred, democratic election is far from being optimal in this respect, and is increasingly becoming the target for fraud. A model was developed to scientifically analyze the present electoral system's insufficiency. It is based on fauceir assumptions. Its calculations enable principles to be developed that optimize the election process, while also revealing the limits of elections in societies growing ever more complex, so that in the end elections have to be replaced by processes similar to what has proved optimal throughout naturally occurring evolution-natural selection.

Key words: Capability quotient, democratic election, fauceir theory, mathematical model, maple software, natural selection

INTRODUCTION

Two fundamental convictions underlie the purpose of this paper: (1) The democratic process is a valuable human endeavor. (2) It is the key process for achieving social improvements. The democratic process involves at least three main functions: making a judgment, holding a ballot, and obediently accepting the result. This article is only about making the proper judgment.

For individuals living in social compounds, nomination of the best leader is crucial to the compound's chances of survival. Vast literature on the mechanisms and outcomes of elections exist, ranging from scientific analysis (Chiao *et al.*, 2008; Rand *et al.*, 2009) to political journalism (Zeleny, 2010). Mathematical models too have been developed (Gelman *et al.*, 2002; Hsu *et al.*, 2005; Dietz, 2006; Jiao *et al.*, 2006; Belenky and King, 2007; Boccara, 2010; Tangian, 2010). However, there is only a small amount of literature that tackles the outcome of an election from the vantage point of the compound's success and competitiveness. This rather objective perspective is assumed by the study presented here.

Next, the fact that social judgment and judging others depends on the self is a truism (Alicke *et al.*, 2005), and several social psychological studies have been conducted that convincingly demonstrated the general unawareness of deficient skills (Kruger and Dunning, 1999) and the feeling about superior humanism and social responsibility (Loughnan *et al.*, 2010). No study has been performed yet on how this biased individual perception affects the outcome of democratic elections. The model presented by this study addresses this problem.

Finally, probably most people would agree that being an outstanding leader involves intelligence, but a formidable intelligence test result does not guarantee an outstanding leader. The concept of intelligence is controversial and still under development. Problems with it arise because every static intelligence scale proves insufficient in an ever-changing social and political context (Flynn, 1984; Flynn, 1987; Teasdale and Owen, 2008), so Gardner's theory of multiple intelligences is just an other step towards a more comprehensive concept (Gardner, 2006). The capability quotient introduced by this study allows a multidimensional evaluation in an evolving environment.

By introducing this new parameter to measure an individual's capabilities to successfully lead a group, the mathematical model presented here adapts a neutral position by statistically estimating the distorted individual perception of these capabilities. From that, conclusions are made about how to improve this process. Next, several ways of manipulating the democratic process are illustrated. Being well aware that all these examples deserve more than a few sentences to be comprehensively covered, the sole purpose of this study is to introduce the model and to give some examples of its usefulness, not to elaborate on historical questions.

Model calculations: The model calculations have been performed in 2010 at the Center for Nephrology and Metabolic disorders with Maple$^{(TM)}$ 10 software on a desktop PC. The source code is provided as supplementary material.

Of note, the final success achieved by the group is the ultimate test of the abilities of its leader. Though objective, this test is cumbersome, time- and resources-consuming, and hardly reproducible, so the best leader should be determined beforehand. The democratic

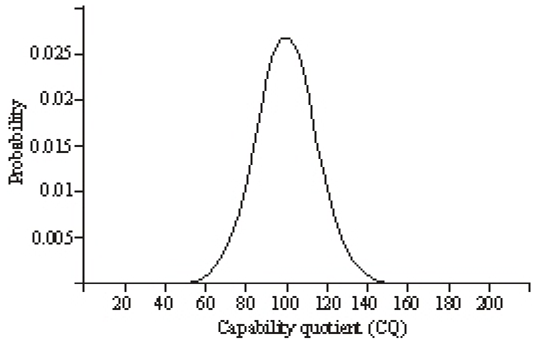

Fig. 1: CQ distribution: The figure illustrates the normal distribution of the capability quotient (CQ) among the population. It is assumed that this distribution is similar to the distribution of the intelligent quotient (IQ) with a mean of 100 and a standard deviation (SD) of 15

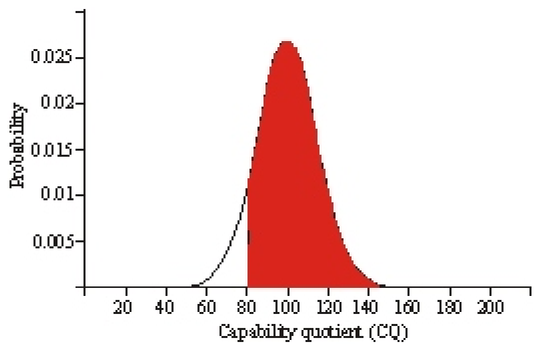

Fig. 2: The CQ distribution mirrored by a low-CQ individual: The normal distribution of CQ can only be measured by an absolute objective method. Each individual perceives a distorted picture of this graph only. This figure illustrates an individual with a CQ of 80 who does not understand the differences in capabilities among members that belong to the orange area. This area includes most of the people of this population

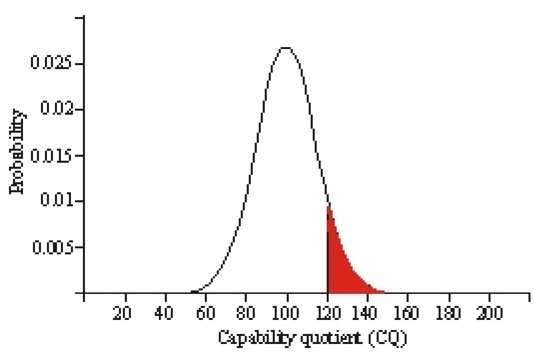

Fig. 3: The CQ distribution mirrored by a high-CQ individual: This figure illustrates an individual with a CQ of 120 who understands the differences in capabilities among members that belong to the not-orange area. This area includes most of the people of this population

process, the model of which is introduced in this chapter, is the typical human device that allows a reasonable prediction. It is not one single property that makes a person an outstanding leader. Rather, we consider leadership qualities as the sum of all the attributes that contribute to an effective leader.

First, we define a scale of leadership qualities in analogy to intelligence. We assume that for each individual a Capability Quotient (CQ) can be determined. That is, the sum of all an individual's leadership qualities divided by the average sum of such qualities. To better handle the figures, this quotient is multiplied by 100. Then the better-than-average leader possesses a CQ greater than 100 while people with a CQ below 100 are not recommended to become leader at all. Next we assume that the distribution of CQ is the same as Intelligence Quotient (IQ). Then the normal distribution's mean is 100 and the standard deviation 15. The left-hand side of the spectrum contains the less capable people while the right-hand side of the spectrum is reserved for the most capable. Hence, the more the leader is selected from the right-hand side of the spectrum, the better the group's outcome (Fig. 1).

Next, this objective spectrum is projected subjectively into each individual's mind, given the empirical knowledge that an individual's ability to gauge an other person's qualities ends at the very point where their own qualities reside on the spectrum, or, in other words, each individual is blind to differences in capabilities better than their own. Again for simplicity, we define a rule of distortion that, depending on one's own position on the spectrum, everything that is left of this position is reflected almost correctly, while all individuals right of one's position on the spectrum are regarded as no better than oneself. In physics this corresponds to a low-pass filter (Fig. 2 and 3).

Finally, given each individual's attempt to select the best possible leader, the leader is selected from all group members that project into an individual's mind as of the same CQ; and assuming that among these people a leader is chosen at random, then the most likely choice is the mean of the distribution rightwards of the individual's own capabilities, which is represented mathematically by the center of gravity's x-value.

Applying this model, we may calculate the probability distribution graph of the selected leader's CQ. The resulting distribution graph is shifted to the right but the mean does not differ too much from the mean of the original distribution (Fig. 4).

DISCUSSION

The model prerequisites: The model is based on the following assumptions:

- Normal distribution of capabilities to lead a group
- An individual's inability to gauge individuals better than self
- Each individual can make the decision independently
- All individuals share an interest in the group's best outcome.

Normal distribution of capabilities: When introduced by Stern (1912) the probability curve of the intelligence quotient showed normal distribution. All the enhancements made to intelligence testing ever since found a normal distribution too, although sometimes with varied standard deviation (Wechsler, 1949; Delaney and Hopkins, 1987). Normal distribution is common to most inheritable human traits that can be measured continuously, such as weight, height, and IQ. The capability to successfully lead a group of present-day humans is a trait closely related to intelligence. Therefore it is plausible to assume a normal distribution of CQ to start with. It can be mentioned, however, that a normal distribution is not required for the model to work, as can be seen after the model has proceeded through several steps.

The blind part of the spectrum: The fact that every individual is incapable of properly evaluating more qualified ones is crucial to this model and to evolution in general. This rule can be induced from trivial experiences and experimental data and deduced from fauceir theory. An obvious example of this would be medieval England after the Norman Conquest. The Normans spoke French predominantly, so every leader after this conquest had to speak French to be successful (Adams, 1969) and hence the non-French-speaking people were incapable of evaluating a leader's capabilities. By the term social projection Allport (1924) succeeded in introducing a scientific term describing the distorted perception of others. Other authors elaborated on the theoretical concept (Asch, 1952; Heider, 1958) or introduced scores to measure this phenomenon (Cronbach, 1955). The first experimental data were provided by Kruger and Dunning (1999) whose findings confirmed their prediction that "the skills that engender competence in a particular domain are often the very same skills necessary to evaluate competence in that domain" in experiments of self-evaluation. By our study this assumption is extended the evaluation of other people.

Deduction is possible from Fauceir Theory that claims that information exchange requires fauceir interaction, which is possible only if special devices have evolved to allow that interaction. So a fauceir that encompasses a certain capability can only be appreciated by fauceirs that can interact with that particular fauceir.

Although there is no doubt about the fact that a distortion of the perception of other peoples capabilities depending on private capabilities exists, and there can be achieved general agreement that capabilities better than one's own capabilities are poorly appreciated, still little is known about how the perception is distorted and what are the factors that in each particular case influence this distortion. This might be the subject of further intensive research. For reasons of simplicity and computational practicability, we have assumed the rightward-blinded spectrum as this would not have implications on the conclusions reached by this study.

The more an individual's capabilities are shifted to the left of the spectrum the more convinced that individual will be that all people are the same. This is the objective reason why socialist sociological theories are more deeply rooted in less-educated people. Being aware of this fact, communist parties sought support among blue-collar workers, whom they called the working class (KPCC, 1961; Honecker, 1967; Ponomarev, 1970).

Independent decision-making: The model holds true only if every member of the group has the same right to choose and to be chosen, which requires that all members of the group know each other well enough. As human memory has only a limited capacity to store such information, this model works only in groups with a limited number of members (Kosse, 1990). This limited number is about 100 people.

Select the the best leader: This assumption holds true if and only if the interests of the individual coincide with the interests of the group. These interests are never identical, which also follows from fauceir theory, but they should go in the same direction for the most part. Otherwise the model does not work, as some individuals would intentionally select a rather incapable leader to better pursue private interests.

Such a strategy reflects the joke: the blind selected the one-eyed king, but the one-eyed have been more clever as they selected a blind king. A similar point of view was recently expressed by an article in 'Time Magazine' where the author quotes a politician who said: some European politicians are "being chosen for their limits rather than their merits" (Robinson, 2010).

Center of gravity: The center of gravity is assumed to be a good approximation as it mathematically reflects the mean x-value of the area-under-the-curve. There might be other methods for calculating the most probable outcome of a random choice, but all would lead to the same conclusion.

Principles to manipulate the outcome: Attempts to manipulate the outcome of an election are as dateless as elections themselves. These manipulations do not essentially result in a less favorable outcome. In fact some

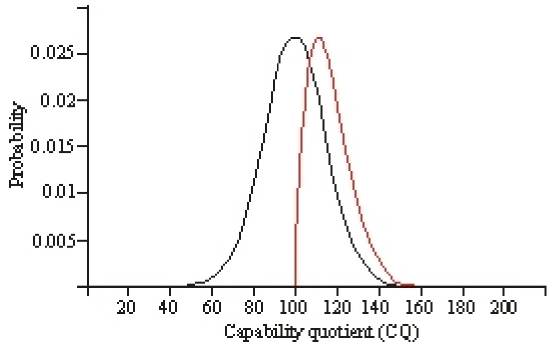

Fig. 4: Outcome of a normal election in the whole population: Given the normal distribution of CQ in this population (black) and the inability to appreciate CQ values better than one's own, the population would select a candidate with a new probability function (red)

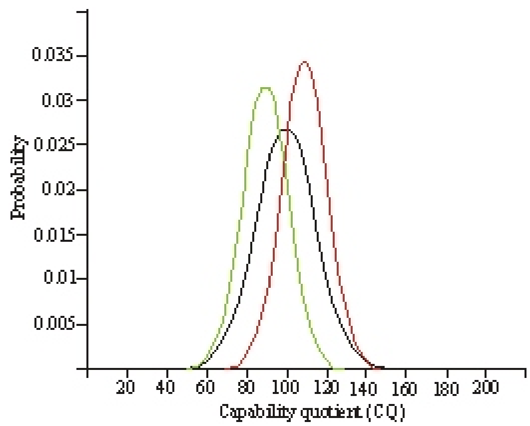

Fig. 5: CQ distribution in a non-representative sample with lower average CQ. If by a random process, a group of individuals with higher CQ (red) is excluded from the original population (black) a new sample population results (green) in which there is a lower average CQ. The new distribution is not necessarily a normal distribution as this example suggests. Depending on the parameters of the elimination process this graph might be more or less distorted

of them have evolved to improve the outcome, but of course, as every new invention, these principles might be used to pursue private interests and to harm the whole group. We may distinguish manipulations that happen before, during and after the actual election. These are called pre-election, influencing judgment, and subsequent improvement steps, respectively.

Pre-election: If not the whole group but only a sample of it is allowed to participate in the election, the outcome can be altered if the sample is not representative of the group in general. The sample can be shifted to the left or to the right of the spectrum accordingly, and a less or more capable set of elected candidates will result (Fig. 5-8).

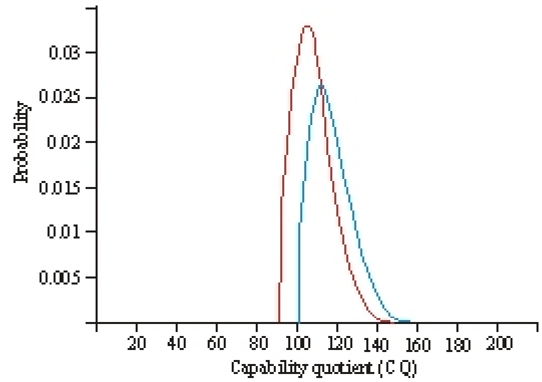

Fig. 6: Outcome of an election in a sample population with a lower average CQ. If the sample population has a lower average CQ, the outcome of an election is a probability distribution with a lower average CQ of elected candidates as compared to the whole population (blue)

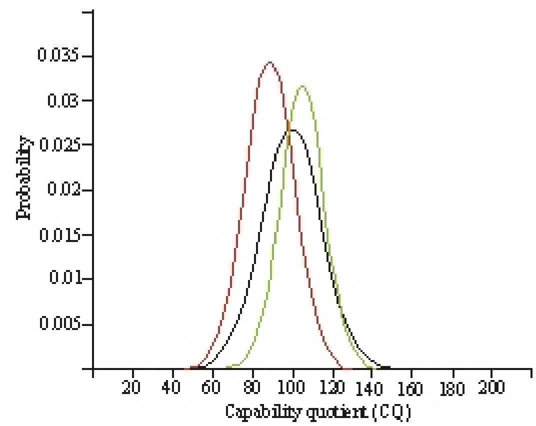

Fig. 7: CQ distribution in a non-representative sample with higher average CQ. If by a random process, a group of individuals with lower CQ (red) is excluded from the original population (black) a new sample population results (green) in which there is a higher average CQ. The new distribution is not necessarily a normal distribution as this example suggests. Depending on the parameters of the elimination process, this might be more or less distorted

Influencing judgment: By contrast to making a judgment rationally and based on one's own experiences, such judgments can be made irrationally only by following a rule of propaganda. This kind of manipulation gains enormous importance if a selection happens in a human society where the members are unable to personally know and learn about a candidate's capabilities. Again these manipulations can be favorable or unfavorable, depending on the candidate encouraged.

Improvement step by step: As the capabilities of an elected person are likely to be slightly better than average (Fig. 4), the outcome of an election can be improved by

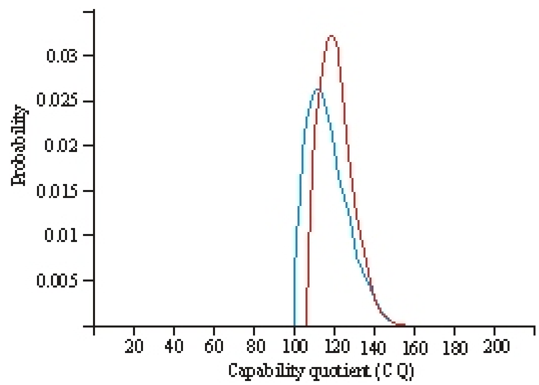

Fig. 8: Outcome of an election in a sample population with a higher average CQ. If the sample population has a higher average CQ, of course, the outcome of an election is a probability distribution with a higher average CQ of candidates as compared to the whole population (blue). Interestingly, the benefit of eliminating the low-CQ subgroup is not as favorable as the elimination of the high-CQ subgroup is unfavorable. Conclusively, the more capable an individual the more important the participation in an election

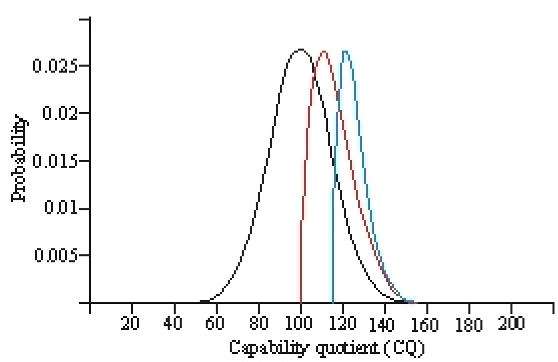

Fig. 9: The outcome of subsequent election steps. If an election is conducted in a general population (black), as we have seen before, a number of selected candidates results whose distribution of CQ is depicted by the red graph. If a subsequent election is now conducted among only these elected candidates, the outcome can be even more improved (blue)

subsequent elections. In a first election held in separate groups, electors are chosen that form a new group in which the next election takes place, and again there is a small benefit (Fig. 9). But as the model illustrates, this cannot be repeated infinitely, because as the benefit by each subsequent election decreases, it converges to zero.

Perspective:

Improved communication: Given the large number of people in a country, an election process in which everybody has the same chances to vote and to be elected is not feasible. Nobody can check millions of voters for their capabilities. But modern communication technology makes it possible to form random election groups, which elect electors that form the next level election group. Given that an election group consists of about only 100 people, only a few steps are necessary to cover a population of several million people.

A subject centered repetitive election: An other problem that became evident by the model can be tackled by modern communication technologies. People are becoming more and more specialized, experts in only a narrow field of interest. These capabilities might be useful for solving questions in their particular field, but it is rather unlikely that these capabilities can be appreciated by others. Thus modern communication technology offers the possibility to form specific election groups for special topics or problems to be solved.

Selection or election: Natural selection and democratic elections are two distinct methods for improving composition and structure of a group of individuals. While natural selection according to evolutionary theory is the dominating process in the realms of animals and plants (Williams, 1966; Darwin, 2009), democratic election is the strategy that humans invented to replace the physical fight for leadership typical of savage tribes or medieval aristocrats. That time, the leaders were almost exclusively male. Because of these historic roots, a study recently confirmed that male voters interpret elections as dominance competitions (Stanton *et al*., 2009). The male's hormonal make-up during an election is equivalent to that during competitive sport. While selection naturally eliminates the least capable from the group, elections work the other way around by choosing the most capable. The election approach is favorable if only one or a few properties are to be tested, while selection is better if the number and interactions of the properties to be evaluated become confusing. For instance, among savage people the ability to protect the tribe played the dominating role, which had been easily tested by fighting competitions. Today, leadership qualities comprise an ever-growing number of capabilities, as reflected by the CQ introduced above. The complexity is comparable to a living organism encompassing a network of thousands of interacting genes that are better controlled by natural selection.

As the model implies each individual better reflects the left part of the spectrum, so it seems quite plausible to find the best possible leader not by choosing directly but rather by subsequently eliminating the less capable from the group of responsible persons.

The impact: By contrast to previous election models (Gelman *et al*., 2002; Hsu *et al*., 2005; Dietz, 2006; Jiao *et al*., 2006; Belenky and King, 2007;

Boccara, 2010; Tangian, 2010) the model presented by this article can analyse the efficiency of the electoral process itself, not just its oucome. This is achieved by a fauceir evolutionary approach which allows the democratic process to be appreciated in the context of a quest to find the optimal social leader.

CONCLUSION

Given the prerequisites and limitations of the model, it allows the following conclusions:

- Democratic elections result in merely mediocre political leaders.
- The improvements that can be achieved by subsequent election steps are limited.
- The main advantage of democratic elections is that they effectively prevent lower-than-average candidates from becoming leaders.
- Substantial and lasting improvements in leadership can be achieved only by improving the average CQ of the entire compound.
- Democratic rules, such as the freedom of the press, have to guarantee that ruling leaders are not allowed to hinder the growth of average capabilities.

ACKNOWLEDGMENT

I would like tothank those contributors to the Maple user forum (http://www.mapleprimes.com/) who anonymously helped to straighten out the mathematical presentation of the problem.

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