

# Calculation in the head: finding the next square and finding the meen

The thing about mental calculation is that the mind is beter at som kinds of calculations than others. The trik to doing mental calculation wel, it to find suitable mental algorithms that one can folow to reech the desired result. Here ar two that i found.

## Finding the next $x^2$

I tryd to see if i cud find som patern in the diference between eech square number, so i started out looking at the first few sqaure numbers: 1, 4, 9, 16, 25. Then i noticed a patern about the diferences:  $4-1=3$ ,  $9-4=5$ ,  $16-9=7$ ,  $25-16=9$ . The diferences increes by 2 eech step. The first algorithm i cud think of was curent result+ $2x+1$ , wher “x” is the factor of the previus sqaure. So, in order to find the next sqaure from  $1^2$  one had to ad the previus sqaure (=1) to  $2x$  (=2) to 1, the result of wich is 4, wich is the next square:  $2^2$ . Then i tested

Testing out the hypothesis in a quasi-empirical way in the hed:

n	$n^2$	hypothesis result
1	1	1
2	4	4
3	9	9
4	16	16
5	25	25
6	36	36
7	49	49
8	64	64
9	81	81
10	100	100

After that i felt certain enof that i had hit upon a corect formula for finding the next square.

Contrary to popular beleef, induction works just as fine in math as it does everywher els.

Then i tryd to see if i cud find a proof, but first, i had to formalize the idea:

$$x^2 = (x-1)^2 + 2(x-1) + 1$$

e.g. as in:

$$4^2 = (4-1)^2 + 2(4-1) + 1 = 3^2 + 2(3) + 1 = 9 + 6 + 1 = 16$$

and then i wanted to see if i cud figur out how to proov it deductivly. I think i found a way:

$$x^2 = (x-1)^2 + 2(x-1) + 1$$

reducing the square to its constituent:

$$x^2 = (x-1)^2 + 2(x-1) + 1$$

factoring in:

$$x^2 = x^2 - 2x + 1 + 2(x-1) + 1$$

factoring in again:

$$x^2 = x^2 - 2x + 1 + 2x - 2 + 1$$

canceling stuf out:

$$x^2 = x^2$$

and thus, by a series of identicals:

$$x^2 = (x-1)^2 + 2(x-1) + 1$$

## Finding the meen

Supose u ar looking at a number of numbers, say: 4,7,13,16,3. How does one eesily find the meen?

The normal method is to ad all the numbers up and divide by the number of numbers aded up.

However, the mind is not good at diviving (long division) unles it is dividing by 2 or som  $10^n$  wher “n” is an integer.

I found this method: First, make som ges as to wat the meen is, call this the hypothesis: h. Then, go thru all the numbers and ad up the diferences. If the sum of diferences is 0, then the hypothesis is corect. If the sum is non-zero, then ad the sum of the diferences divided by the number of numbers to the hypothesis, and that is the corect meen.

E.g., using the numbers abov. Suppose we ges that the meen is 10. We then go thru all the numbers:

$$4 - 10 = -6$$

$$7 - 10 = -3 \quad \text{sum: } -9$$

$$13 - 10 = 3 \quad \text{sum: } -6$$

$$16-10=6 \quad \text{sum: } 0$$

$$3-10=-7 \quad \text{sum: } -7$$

so, the ges was rong. We can get the corect number by ading the sum divided by the number of

numbers (=5), so, we ad  $\frac{-7}{5}=-1.4$  and get  $-1.4+10=8.6$  wich is the corect meen.

We chek by calculator:  $4+7+13+16+3=43$  and  $\frac{43}{5}=8.6$  . The method seems to be working.

I tested it out with a few other randomly chosen sets of small numbers and it worked everytime (inductiv evidens again). That convinced me that i had a working algorithm.

Wen i found this algorithm, i didn't no how to proof it deductivly, so, i spoke with a mathmatician.

He ofered this proof. First, let's formalize things. This is the definition of "meen":

$$\frac{\sum_{i=1}^n n_i}{n} = m, \text{ wher "m" is the meen.}$$

and my theorem is:

$$\frac{\sum_{i=1}^n n_i - h}{n} + h = m, \text{ wher "h" is the hypothesized meen.}$$

first, we use this rule:

$$\sum a \pm b = \sum a \pm \sum b$$

and get:

$$\frac{\sum_{i=1}^n n_i}{n} - \frac{\sum_{i=1}^n h}{n} + h = m$$

then we use this rule:

$$\sum a \cdot c = c \sum a, \text{ wher "c" is a constant}$$

and since "h" is a constant, we get:

$$\frac{\sum_{i=1}^n n_i}{n} - h \frac{\sum_{i=1}^n 1}{n} + h = m$$

wich is equivalent with (altho i'm not sure wy precisely):

$$\frac{\sum_{i=1}^n n_i}{n} - h \frac{n}{n} + h = m$$

and then since stuf cancels eech other out, we get:

$$\frac{\sum_{i=1}^n n_i}{n} = m$$

and therfor, by a series of identicals:

$$\frac{\sum_{i=1}^n n_i - h}{n} + h = m$$

follows.

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The reason this algorithm works better for the mind than the normal sum/divide method, is that it is much easier to divide in the mind a small number (the sum of differences) by  $n$  and add that to some

number, than it is to divide the sum of all the numbers. E.g., it is easier to divide  $\frac{-7}{5}$  and add that

to 10, than it is to divide  $\frac{43}{5}$ .