

Classical predicate logic and empty domains

There are a few reasons to think that classical predicate logic (CPL) isn't a true logic (let's keep that vague). Specifically, an 'argument' much like the one below seems to show that CPL gives absurd results.

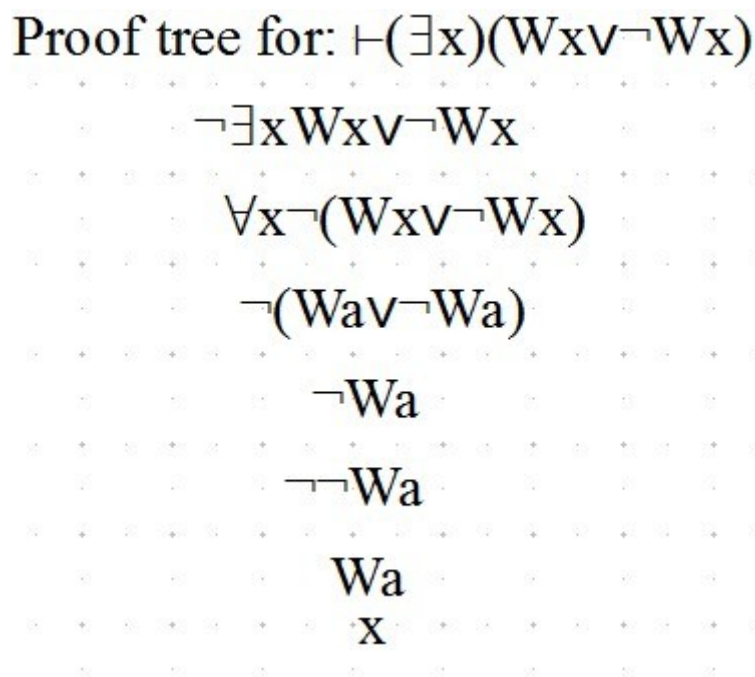
Domain, x = unicorns

Wx = x is white

Thus, 1. There is a unicorn, x , such that x is white or it is not the case that x is white.

$\vdash (\exists x)(Wx \vee \neg Wx)$

Now, this 'argument' is valid, that is, the conclusion is necessarily true according to CPL. The proof tree for the 'argument' is:



So, we have proved that there is a unicorn which is either white or not white. But there is no unicorn. So something went amiss. The usual solution is to allow for empty domains (free logic), but this doesn't seem necessary to deal with such 'arguments' as the one above. An easier solution is just to make the rule that one cannot use empty domains in an interpretation. Without this such 'arguments' as the one above doesn't work. Sure, the wff is still a tautology, but that hardly matters. Wff's are not true or false (if that even makes sense to say). Only an interpreted wff (which is a sentence) has a truth value (or expresses a proposition that has one).

I I prefer not to call arguments (or 'arguments') from no premises, "arguments"

Still, one may ask:

What about the empty possible world? Shouldn't it be true in that world that the any chosen domain is empty?.

The answer is “No”. Even the empty world has objects in it: Abstract objects exist in all possible worlds. There is no world which has no objects in it at all. So, we cannot use the above wff to get a conclusion that is false in the empty world.

But there is another possible problem. Suppose we are reasoning about the empty world. Then we make an 'argument' with the above 'argument form' and interpret it as:

D, x = things

Bx = x is green

Thus, 1. There is a thing such that it is green or it is not the case that it is green.

$\vdash. \exists x Gx \vee \neg Gx$

But, the empty world has only abstract objects in it, and none of them are green or not green! So, have we reached a false conclusion? No. This interpretation does not actually produce an argument. What it does produce is a meaningless collection of words. It is meaningless to say things like “There are no green abstract objects”, not false. But many people do think it is false. So, we cannot like this produce a problematic argument for CPL.