

**Abstract.** Modern logic is undergoing a cognitive turn, side-stepping Frege's 'anti-psychologism'. Collaborations between logicians and colleagues in more empirical fields are growing, especially in research on reasoning and information update by intelligent agents. We place this border-crossing research in the context of long-standing contacts between logic and empirical facts, since pure normativity has never been a plausible stance. We also discuss what the fall of Frege's Wall means for a new agenda of logic as a theory of rational agency, and what might then be a viable understanding of 'psychologism' as a friend rather than an enemy of logical theory.

**Keywords:** Psychologism, Psychology of reasoning, Dynamic logic, Interaction, Self-correction, Cognitive science.

## 1. Drawing borders

Logic arose in Antiquity from two sources: the study of real argumentation in the dialectical tradition, and that of axiom-based proof patterns organizing scientific inquiry. Over the centuries that followed, the discipline turned abstract and eventually, profoundly mathematical. Is logic still about human reasoning? Or is it, as Kant and Bolzano said, about an abstraction in the realm of pure ideas? In the latter vein, logical consequence is an eternal relationship between propositions, firmly cleansed from any mud or blood stains, smells, or sounds that human inferences might have — and therefore also of their colours, and tantalizing twists and kinks. Put in another way, do the empirical facts about human reasoning matter to logic, or should we just study relationships between proof patterns, and the armies of them that we call formal systems, in some eternal realm where the sun of Pure Reason never sets? Most logicians think the latter — and accordingly, the precise relation to reasoning practice is left open. Universities should just hire logicians for their intrinsic cultural value, and no questions asked: in Tennyson's immortal words, "theirs not to reason why". Of course, if pressed, many logicians and philosophers would say that logic is normative; it describes *correct* reasoning. People would be wise to follow its recommendations — but so much the worse for them, if they do not. In fact, did not

Wilkie Collins, one of the first mystery writers<sup>1</sup> say it all, in the immortal words of the family butler Gabriel Betteredge (*The Moonstone*, 1868):

*“Facts?” he repeated. “Take a drop more grog, Mr. Franklin, and you’ll get over the weakness of believing in facts! Foul play, sir!”*

The divide between logic and human reasoning is enshrined in Frege’s famous doctrine of ‘Anti-Psychologism’, claiming that human reasoning practice can never tell us what is a correct conclusion. Indeed. So what? I myself find this easy line of demarcation a sign of intellectual poverty, rather than ‘honest’ splendid isolation.<sup>2</sup> Frege wrote in the days when serious modern psychology was taking off, resulting in the wonderful work of Helmholtz ([15]), and Heymans on mathematical reasoning and the origin of mathematical concepts, which inspired the mathematical theory of transformations and invariants. It suffices to compare the self-centered formal philosophy of Frege’s prose, uninformed by what was going on outside, with the much more informed and wide-ranging text of the pioneering psychologist Wilhelm Wundt, which shows awareness of modern logic.<sup>3</sup> Even so, Wundt definitely failed to see the great power of mathematical abstraction in logic, and the advantages of stepping aside from practice in order to gain strategic depth. But Frege seems blind to the creative impulses that might come from at least thinking about the ‘best practices’ of humans, and psychology as the scientific chronicler of these.

Anti-Psychologism<sup>4</sup> is still defended today with a great deal of dogmatic fervour, not just in logic, but also in philosophy in general. When I mention what seem to me interesting and suggestive experimental facts about human reasoning that are coming to light these days, I am often lectured by colleagues who tell me that I have not understood the first thing about the essence of logic — or philosophy for that matter — which is about correctness and mathematical relationships between propositions, not about what goes on ‘outside’ in that frustrating world populated by dimwits and non-mathematicians. It is good that I had tenure when I decided to come out (however modestly) in favour of facts!

<sup>1</sup>Detective novels are an early form of applied logic which resonates in human practice.

<sup>2</sup>My initial plan for this paper was a modern reappraisal of Frege’s argument, until I realized this would mean falling into the very trap one wants to avoid: discuss links to the outside world as an indoor issue.

<sup>3</sup>Wundt criticized Boole’s system of propositional inference for its great distance from practice in ways that are still relevant today.

<sup>4</sup>Anti-Psychologism is an abstract stance like Anti-Americanism. It allows logicians to associate with individual psychologists on friendly terms provided the latter colleagues do not attempt to preach their scientific insights.

Now comes my simple declaration of faith. Logic is of course not experimental, or even theoretical, psychology, and it approaches human reasoning with purposes of its own. And a logical theory is not useless if people do not quite behave according to it. But the boundary is delicate. And I think the following should be obvious: if logical theory were *totally disjoint* from actual reasoning, it would be no use at all, for whatever purpose!

In the rest of this essay, I attempt to chart the much more complex actual relationship between theory and human practice in logic: sometimes exasperating, but always fertile.<sup>5</sup> My model is the presentation of logic in the Handbook chapter [29].

## 2. Logical systems and human behaviour

Let us be clear first on some form of demarcation. An inference pattern like

Modus Tollens      from  $A \rightarrow B$  and  $\neg B$ , it follows that  $\neg A$

is valid whether we want it or not, and not even a democratic UN endorsed referendum on our whole planet would change that. So much is true. But where is the necessary opposition to practice? We humans are not smelly noisy irrational images of a world of mathematical beauty: as Plato thought.

<sup>5</sup>Many other disciplines are in the same boat sailing a tortuous course in between normative theory and descriptive practice, including *probability theory*, *game theory*, or even *linguistics*. For the delicate position of game theory, cf. the perceptive entries by Camerer, Rubinstein, and others in [14]. As for the status of probability theory, this would deserve a separate essay — but let me just say this. The situation here is like logic in that there is a tension between the correct rules of the basic calculus and the ‘failures’ that are reported when human reason with uncertainty, often accusing subjects of blatant errors, such as converting the conditions in conditional probability. ([11] provide some counter-evidence of sane reasoning, even with young children.) But there is also an established tradition acknowledging that the rules of the core calculus cannot be tested separately from the choice made by the subjects of a *probabilistic model* for the situation they imagine themselves to be in. The analogue for this in the case of logical reasoning would be a careful study of formal options for choosing and managing qualitative models — as provided in the re-examination of the Wason Card Task in [23]. But cf. Section 2 below for some reservations on this ‘free parameter’ when invoked indiscriminately. In any case, probability theory is unlike logic in that it has ‘internalized’ the tension between dual views of its core notion, studying objective frequency-based probability in tandem with subjective probability as degrees of belief. In this light, the oldest explanations of probability, as in Johan de Witt’s famous treatise “Waerdije” from 1671, in terms of observable betting odds, are a pretty sophisticated compromise. Here, a subjective notion is made measurable by observable transactions. What would be the analogue of the usual economic notions of ‘revealed belief’ or ‘revealed preference’ in the case of logic?

We, too, are designed by Nature according to those laws, so there is no *a priori* conflict between what we do and logical norms.

Indeed, and more importantly, if our observed practice ‘diverges’ from some logical norm, what does that *mean*? Let us first state some common distinctions. Before we can reason at all, we must have some model of the situation that we are supposed to be discussing, and this includes *construing the relevant scenario* of facts and events, while representing the information that these convey. This construction can be conscious, or hard-wired into our neural nets, but it represents a pre-processing phase before inference can take place as a process of transforming information. Next, on these information structures, we can perform an inference, but there may be a choice of different modes appropriate to the task at hand: classical or constructive or ‘linear’ in mathematics, classical or non-monotonic in problem solving, and so on. Given the options in this double scheme of *representation + transformation*, many, if not most human practices can be explained in a manner consistent with logical theory. Indeed, it is so hard to find genuine clashes, that our worry ought to be the *lack* of divergence. The representation + transformation scheme is almost too good as a ‘protective belt’ for logical theory versus practice: it can fit anything.<sup>6</sup> The resulting immunity for logic would not please Popper, and even worse than that: it seems boring at times.<sup>7</sup> In my view, I would rather let logic benefit from the contact, *learning* from direct confrontation with practice. To do so, one needs to move closer to the goal of providing more direct and faithful mathematical renderings of what seem to be stable reasoning practices. And in fact, there is no need to just speculate about all this, because it has happened many times already, and it still is happening all around us. Thus, instead of spinning further *a priori* philosophies of logic<sup>8</sup>, let us take a broad look at some real historical developments over the past decades.

<sup>6</sup>Indeed, the representation-transformation scheme for analyzing logical practice comes dangerously close to broad immunization strategies for theory versus practice such as the well-known linguistic distinction between ‘competence’ and ‘performance’.

<sup>7</sup>One popular view of logic is as an ‘arsenal of formal systems’ from which an applied logician can choose given any conceivable task at hand. I find this take-out menu idea static and insensitive, while it identifies logic with just formal systems: a view criticized as ‘system imprisonment’ in [27].

<sup>8</sup>Current textbooks in the philosophy of logic have a thematic agenda mostly free from any information about actual developments modern logic, making them conserve the status quo of at best the 1960s.

### 3. Logical theory already follows practice!

In terms of the above division of labour, there are two major issues: one of representing information, and another of the processes that transform these representations. Both affect contemporary logic through various channels.

For a start, much of what is often called ‘philosophical logic’ has actually been about representations of characteristic structures in language and thought that go beyond the bare minimum provided by standard first-order logic.<sup>9</sup> To mention just two examples that have led to major research areas of their own, Prior’s work in the 1950s on time and temporal reasoning introduced *temporal structure* into logical models to account for the tenses and time-related modalities of actual use. And equally famously, in the 1960s, Lewis and Stalnaker introduced *comparative orderings* of worlds or situations, with the key logical notion of a standard conditional  $A \Rightarrow B$  saying essentially that the minimal worlds in the relevant ordering where the antecedent  $A$  is true also have the consequent  $B$  true. Benchmarks for ‘correctness’ of such logical accounts were diverse: the analysis of philosophical arguments stated in natural language, sometimes *a priori* conceptual analysis per se, but they definitely also included actual ordinary usage. For instance, conditional reasoning is close to the ‘irrealis’ mode that humans engage in when considering situations beyond the immediate here and now.<sup>10</sup> Following up on this phase, but still very much in the same spirit, came the work in *logical semantics of natural language*, with its account of many expressions beyond standard logical formalisms, such as generalized quantifiers, moving much closer to actual reasoning patterns. This resulted, amongst others, in much richer accounts of information states for language users, with discourse representation theory as a well-known example. Finally, modeling more realistic information states in logical terms has been a hallmark of *Artificial Intelligence* (according to one Dutch author, the continuation of philosophy by computational means), especially in its guise of analyzing ‘common sense reasoning’.

Even though none of these logical theories involved actual appeals to psychological experiments<sup>11</sup>, pioneers like Prior, Lewis, or Hintikka did ap-

<sup>9</sup>We refer the reader to [7] for general information on this area.

<sup>10</sup>Philosophical logic is often taken to be the mind police of ‘formal language philosophy’ — whereas ‘natural language philosophy’ was close to observations about actual human behaviour from the start. But empirical observations about usage do play a crucial role in both of these great branches of analytical philosophy.

<sup>11</sup>Some semantically oriented philosophers even appealed with unholy pride to *armchair intuitions* as preferable to actual usage.

peal at crucial points to intuitions shared by ordinary language users — and not just a priori ideas of validity.<sup>12</sup> In recent years, real-life links with actual experiments have also started emerging: cf. Section 4 below. This trend toward more realistic modeling of information is in fact very natural, and it fits well with the *expressive face* of logic, as a theory of models and definability. Textbooks of logic do not normally describe the field in this way, but one might summarize this trend toward richer modeling in terms of a set of broad ideas: families of worlds, temporal perspective, minimization along orderings, and at the level of syntax, issues of text coherence and incremental construction of representations.<sup>13</sup>

The next step is to actually use these richer representations for logical purposes, and here again, logical theory has in fact been greatly influenced by observing practice. The idea that logic is about just one notion of ‘logical consequence’ is actually one very particular historical stance. It was absent in the work of the great pioneer Bernard Bolzano, who thought that logic should chart the many different consequence relations that we have, depending on the reasoning task at hand. A similar rich view of the subject matter of the discipline is still found in the work of Mill, and especially, C. S. Peirce, who studied combinations of deduction, induction, and ‘abduction’: all of them highly relevant today. This sort of variety got further impetus in the 1980s with ‘non-monotonic logics’ for default reasoning coming from AI, which model more closely how humans would approach problem solving or planning tasks. The important thing in this logical theory of consequence is not some sort of catalogue of inference rules (‘non-monotonicity’ is just a symptom, not a diagnosis), but rather, the main lines behind specific proposals, such as again minimization along various orderings in the conditional style. But many further broad ideas in the literature reflect human practice, such as the crucial role of ‘resources’ in establishing inferences, as happens in linear and general sub-structural logics.<sup>14</sup>

To me, however, the most striking recent move toward greater realism is the wide *range of information-transforming processes* studied in modern logic, far beyond inference. As we know from practice, inference occurs intertwined with many other notions. In a recent ‘Kids’ Science Lecture’ on logic for children aged around 8, I gave the following variant of an example

<sup>12</sup>By now, in the semantics of natural language, this love of facts has even gone all the way toward actual statistical corpus research.

<sup>13</sup>Another example of this wider range is modern work on graphical versus symbolic representations in human reasoning, logic and computer science, as found in [2, 17].

<sup>14</sup>By way of contrast, compare the weird, but nevertheless often-cited view of modern logic as just a catalogue of ‘deviant logics’ in [13] and subsequent publications.

from Antiquity, to explain what modern logic is about:

You are in a restaurant with your parents, and you have ordered three dishes: Fish, Meat, and Vegetarian. Now a new waiter comes back from the kitchen with three dishes. What will happen?

The children say, quite correctly, that the waiter will ask a *question*, say: “Who has the Fish?”. Then, they say that he will ask “Who has the Meat?” Then, as you wait, the light starts shining in those little eyes, and a girl shouts: “Sir, now, he will not ask any more!” Indeed, two questions plus one inference are all that is needed. Now a classical logician would have nothing to say about the questions (they just ‘provide premises’), but go straight for the inference. In my view, this separation is unnatural, and logic owes us an account of *both* informational processes that work in tandem: the information flow in questions and answers, and the inferences that can be drawn at any stage. And that is just what modern so-called ‘dynamic-epistemic logics’ do! (See [32] and [30].) But actually, much more is involved in natural communication and argumentation. In order to get premises to get an inference going, we ask questions. To understand answers, we need to interpret what was said, and then incorporate that information. Thus, the logical system acquires a new task, in addition to providing valid inferences, viz. systematically keeping track of changing representations of information. And when we get information that contradicts our beliefs so far, we must *revise* those beliefs in some coherent fashion. And again, modern logic has a lot to say about all of this in the model theory of updates and belief changes.

Moreover, in doing so, it must account for another typical cognitive phenomenon in actual behavior, the interactive *multi-agent* character of the basic logical tasks. Again, the children at the Kids’ Lecture had no difficulty when we played the following scenario:

Three volunteers were called to the front, and received one coloured card each: *red, white, blue*. They could not see the others’ cards. When asked, all said they did not know the cards of the others. Then one girl (with the white card) was allowed a question; and asked the boy with the blue card if he had the red one. I then asked, before the answer was given, if they now knew the others’ cards, and the boy with the blue card raised his hand, to show he did. After he had answered “No” to his card question, I asked again who knew the cards, and now that same boy and the girl both raised their hands ...

The explanation is a simple exercise in updating, assuming that the question reflected a genuine uncertainty. But it does involve reasoning about what

*others* do and do not know. And the children did understand why one of them, the girl with the red card, still could not figure out everyone's cards, even though she knew that they now knew.<sup>15</sup>

The card scenario also points at a wider setting beyond single steps of getting an answer, drawing an inference, and maintaining coherent beliefs. Communication, argumentation, or games involve *longer-term* patterns of interaction, where the reasoning steps serve some over-all *purpose*. Most logical activities are in fact interactions between several agents, from question answering to argumentation or merging beliefs from different sources. This is the point of modern *logical dynamics* ([26, 29]).<sup>16</sup>

#### 4. From the cognitive sciences to logic, and back

Admittedly, the link to ‘reality’ in all these developments is not one with experimental psychology or neuroscience. The love of facts can be very Platonic, without going for major public sources of established information. Logicians analyzing natural language, or computer scientists modeling ‘common sense’, tend to go by their own intuitions, anecdotal evidence from colleagues, email surveys of sometimes surprising naiveness, and other easy procedures that avoid the laboratories and statistical packages of the world of careful experimental design. But even so, experimental evidence is relevant, in that these theories can be, and sometimes are, modified under pressure of evidence from actual usage, even when it comes through these home-grown sources. Moreover, there has been a growing body of more serious literature connecting up logical research with experimental psychology. I cannot discuss the evidence for this in detail, but it may suffice to list a few reputable sources here, without further elaboration.

For a start, logical semantics of natural language has long been an interface with psychology, witness the chapter by Steedman in [31] (a handbook on Logic and Natural Language which borders on psychology at many places), or the work on ‘natural logic’ in [9], the work relating dynamic default logics to conditional reasoning highlighted in the discussion of Kahneman & Tversky’s findings in [34], the monograph [22] on a decade of research

<sup>15</sup>I had been warned by a colleague in psychology that the experiment might not work, but at least the NEMO children seemed to have quite a reasonable ‘theory of mind’, as this iterated knowledge is called.

<sup>16</sup>Many of these broader informational processes, and their treatment in logic, show influences from computer science. We will not pursue this theme here, but we do note the growing importance, also in logic, of computational structures in behaviour, and related issues of *computational complexity*.

into visual versus symbolic reasoning, [33] on default reasoning and natural language understanding, [5] on psychologically plausible models for revision of beliefs and goals, [6] on the formation and maintenance of collective intentions, and many other sources with a wide variety of themes. Another rich emerging interface is that between psychology and non-monotonic logics, witness the 2005 Synthese volume edited by Hannes Leitgeb & Gerhard Schurz on ‘Non-Monotonic and Uncertain Reasoning in Cognition’ ([18]), as well as a recent convergences between default logics and neural nets (see below). A masterful summary of connections with neuroscience is in [1].<sup>17</sup> Thus, significant emerging contacts between logic and psychology are easy to trace in the current literature — even though a *communis opinio* still holds that there are none.

**From cognitive science to logic** My own interest in all this are several larger questions, which all concern the functioning of reasoning in some broader sense. What I find intriguing about our cognitive behaviour are a number of features that seem to call for a richer notion of a ‘logical system’. One of them is the *situatedness* of reasoning, which has been noted by philosophers and experimental scientists alike. It involves both the ‘embodied’ nature of cognition, and the role played by successful linkages between those bodies and their physical environment. In particular, actual reasoning seems ‘situational’ in the sense of [3], involving both *observation* and inference. Recall the earlier restaurant scenario with both questions and inferences. The questions are of course the relevant observations — and any observation comes from a question to Nature. Accounting for this in a logical system involves mixtures of two crucial logical notions: ‘model checking’ and consequence. Though logicians would usually put these in separate compartments, I am intrigued by just how various sorts of *mixtures* of logical tasks work. But continuing right on with cognitive reality, there is another major information source that we usually do not take into account in logical systems, viz. our *memory*. Modern linguistic theories have begun to take this crucial human capacity seriously (cf. the paradigm of ‘data-oriented parsing’, [4]). A traditional theorem prover approaches every new problem like a *tabula rasa*, but we humans do not - not even committed logicians. With experience, we accumulate a stock of understood sentences, comprehended situations, and solved problems, and naturally, as we are confronted with a new task, *two* processes kick in: pattern recognition and *memory*

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<sup>17</sup>Compare also the adjoining tradition of cognitive studies into reasoning with uncertainty, where major researchers from both sides participate: witness [12], [24] or [25].

*search* to find related solved problems, and rule-based analysis. This combination is a much better model for actual reasoning than just proof search, even in pure mathematics. And modern theorem provers also keep statistical records of past performance to aid in new tasks in various ways. The interplay of memory-based search and inference rules in solving problems seems a wonderful challenge to logicians. True, I would expect that proof systems based on this architecture would lead to some exciting new theory only if merged with statistics: but what would be wrong with that?

Next, of all the cognitive phenomena that clamour for attention, I find two of particular logical interest. While the above systems tend to model mature performance in steady state<sup>18</sup>, perhaps the most striking cognitive phenomenon is *learning*. How do we come to learn logical inference, a set of skills which only comes in stages ([20])? And like in modern linguistics, should not ‘learnability’ place a constraint on systems that we design? Formal learning theory (itself an off-shoot of logical recursion theory) has interesting things to say here, but I have never seen any explicit more full-fledged account. The other thing that strikes me again and again is the *diversity* of cognitive agents. There is not one idealized norm for behaviour: some people do better than others on certain tasks, and we manage to co-operate quite well - and we even manage to orchestrate ‘cognitive partners’ that can be quite dumb, such as simple machines, into our total symphony of activities.<sup>19</sup> Finding parameters for diversity of behaviour inside logical systems is again an intriguing issue<sup>20</sup>, and one which again underscores the interactive competences that we have.<sup>21</sup>

Note that this broader agenda is not at all hostile to traditional logic, not even to its standard emphasis on *logical systems*. Such systems are a good focus for research, provided we see them for what they are (I owe this point to Barwise and Etchemendy): *models* of certain styles of reasoning, with a certain expressive power in representing information. In particular, even logical talent shows cognitive diversity, and comes in two flavours. Some colleagues create new systems by looking at the world outside and modeling new phenomena, while others study mathematical properties of and relations between such systems, thus ensuring the flow of information within the field.

<sup>18</sup>Maintaining a searchable inferential memory allows for ‘growth’ in performance.

<sup>19</sup>A related area where this diversity is acknowledged is the literature on ‘bounded rationality’ in game theory.

<sup>20</sup>Cf. [19] for a first study of agent diversity in the context of dynamic-epistemic logic.

<sup>21</sup>For further broad ‘architectural issues’ in the design of logical systems, cf. [27].

**From logic to cognitive science** There are also reverse influences, from logic to cognitive science. Much publicity has attached on work in the ‘psychology of reasoning’ ([35]) showing that people do not think according to the rules of logical calculi. And publishing papers like that never fails to produce a thrill, even these days. But the time seems ripe, also in this opposite direction, for some more mature contact. First, as we have already noticed, the ‘facts’ of human behaviour as found in experiments need to be interpreted, and then, the more startling heralded ‘divergences’ may be questionable. Also, there is the issue of selection. A psychologist, not very well-disposed toward logic, once confessed to me that despite all problems in short-term inferences like the Wason Card Task, there was also the undeniable fact that he had never met an experimental subject who did not understand the logical solution when it was explained to him, and then agreed that it was correct. Why should the latter slightly longer-term ‘reflective fact’ be considered less of a cognitive reality than the former? And more generally, existing experiments in the psychology of reasoning are just a few islands in an ocean of practice. Pioneering experiments are like coral reefs, in that they accumulate decades of follow-up, but the Pacific archipelago remains. Logical theories should then be quite welcome here, as a means of deriving predictions, even if they turn out refuted.<sup>22</sup> Indeed, the above-mentioned logical theories of inference, update, and interaction all suggest interesting testable hypotheses about human behaviour, and one could easily imagine a world where a logician who has created a new logical system does two things instead of one: like now, submit to a logic conference, usually far abroad, but also: telephone the psychologist next door to see if some new nice experiment can be done. And finally, going yet a bit further, I would think that logic can also contribute to a better understanding of how humans form and maintain representations of scenarios and their relevant information, the stage prior to any significant processing. What this would involve is a broadening of current ‘model theory’ to a ‘theory of modeling’.

Clearly, these are all phantasies and expectations. But there is a growing body of work of responsible contacts. Some evidence may be found in the recent *Topoi* issue “Logic and Psychology” ([16]), devoted to current interfaces. In some 150 pages, this volume contains a surprising set of contributions on belief revision, default reasoning, numerical reasoning, natural language interpretation, conditional reasoning, and cognitive evolution, with

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<sup>22</sup>Cf. Popper’s notion of ‘search-light theories’ needed to discover *relevant* experimental facts. Similar points have been made in neuro-science, where a hyper-modern measuring device is no guarantee for asking interesting questions.

extensive connections between logic, linguistics, game theory, and cognitive psychology and brain research.<sup>23</sup>

**An illustration: logic and intelligent interaction** The above set of issues may seem like a mere wish-list of things to be done, and then probably by *others*. But many of these themes occur concretely in current research on dynamic logics for information flow and games, an area which I am involved in myself. Let's go back to the children in NEMO. The above plays there, right before our very eyes. Intelligent behaviour involves multi-tasking: not just logical inference as the measure of all things, but the ability to ask questions, and get the right information out of answers. In doing so, we must represent information about our current situation, and keep that well-attuned through appropriate updates.<sup>24</sup> Thus, both the *situatedness* of reasoning and the *mixes of logical tasks* (inference, evaluation, update) discussed before must be — and are — addressed by dynamic epistemic logics. Moreover, in doing so, ideas from cognitive psychology are percolating. What we have taken from the philosophical logic tradition is the idea that we can get by with the major attitudes of knowledge and belief, and how these change over time. But by now, people have noticed that a *richer set of cognitive attitudes* may be involved, from neutral ‘entertaining’ of propositions to warm-blooded ‘belief’. Also, they have started investigating the richer entangled dynamics of preferences, goals, and intentions. And focusing on the belief revision in particular, we have a natural connection with the more general phenomenon of learning. Belief revision policies are like learning strategies, and their success should be discussed, not in terms of philosophical armchair intuitions, but in terms of known methods of assessment in learning theory. Now once more, back to the children! Clearly, any classroom population shows *diversity* in styles and talents, and logical systems can even help us chart its sources. We see different powers for agents

<sup>23</sup>The list of contributions includes: d'Avila Garcez, Gabbay & Woods on neural net-based mechanisms for abduction, deduction, and induction, Benz & van Rooij on optimal communicative behaviour among several agents in cooperative dialogue, Castelfranchi & Lorini on surprise as a trigger for belief revision, Clark & Grossman on numerical reasoning in the brain as disjoint from language understanding, Knauff on logical reasoning in the brain and its interaction with language, vision and other modules, van Lambalgen & Stenning on conditional reasoning modeled by default logics and mental pathology, Leitgeb on what state of mind constitutes a ‘conditional belief’ drawing on both philosophical epistemology and cognitive psychology, Politzer on the state of the art in cognitive studies of conditional sentences, and Wind Cowles, Walenski & Kluender on the role of topic and focus in textual coherence.

<sup>24</sup>Current dynamic update logics even model real physical changes in the world.

of inference and computation, of policies for belief revision, or even of plain *memory* capacity — and these can be studied in our logics (cf. [19]). Cognitive psychology would pose interesting challenges to this sort of research, since we would like to find logical reasons for the undeniable success of simple knowledge — and ignorance-based algorithms such as those of [10].

Finally, let me add a further aspect of much current research, its move toward multi-agent interaction and *group phenomena*. Much cognitive behaviour resides in how we interact with others. Some researchers find a lonesome logical inference a paradigmatic cognitive peak experience. I myself would find a heart-to-heart conversation or a committee meeting at least as striking as a display of what makes us intelligent. And indeed, the trend is clear. In addition to individual knowledge, logicians study common knowledge and other collective attitudes in groups, they look at belief merges in groups of agents, rather than single revisors, and they cross over into studies of argumentation ([8]). This interest in intelligent interaction gets a very pregnant form in current interfaces between logic and *game theory* ([28]). There one also studies the longer-term strategies which agents have in response to subsequent moves by others, combining perspectives from logic, computer science, and economics. This leads to the longer term of cognitive behaviour over time. But this too, should be on the cognitive agenda — and in fact, studies like [21] show what natural mixes arise then, even with philosophical epistemology. Interestingly, game theory has made its cognitive move in the early 1990s with the emergence of ‘experimental game theory’, an enterprise where even Nobel Prize winners like Reinhard Selten are involved. Maybe, logicians are just catching up. I find the topic of *games and interaction* between different agents also interesting as it does not favour either side. Admittedly, this stance is still marginal in logic, while traditional psychology still largely studies single-agent achievements.

## 5. A new psychologism

So, what ‘ideology’ follows from all this? In line with the title of the present issue, I would have no problem to subscribe to a New Psychologism. But the slogan may be much less provocative than it sounds. What the above observations mean to me is that there can and should be a richer conception of logic than what we have so far, inspired by confrontation with the empirical facts. This confrontation should be taken in the appropriate sense. Advertising ‘mismatches’ between inferential predictions of logical systems, usually without proper attention to the modeling phase, and what is observed in experiments with human subjects seems entirely the wrong focus

to me — not to mention the fact that it is silly and boring. The much more interesting issue is to avail ourselves of broad psychological insights as to what is involved in how people really reason. My examples show how well logic has been able to absorb such insights into richer systems, and much more can be expected. Now, this is truly spoken like a logician, I would say. My interest is in richer logical *theory*, closer to the facts. In what sense can this be called ‘psychologism’? Well, compare the evolution of the term ‘physicalism’. Nowadays, that is not the 18th century claim that everything is just moving particles and their collisions, but the idea that mental behaviour can be described using the highly sophisticated abstract concepts of modern physics. Likewise, ‘human behaviour’ as brought to light by psychology is not just a set of protocol sentences in simple-minded experiments, but a hierarchy of description levels, ranging from plain observable facts to sophisticated higher-order descriptions. Viewed that way, the fit with logical theory becomes much more plausible, in both directions.

So, maybe a ‘psychologist’ stance is innocuous and self-evident? In my experience, changing even the slightest bit of the standard logical agenda meets with determined resistance from many logicians, whether on the mathematical or philosophical side. Change is perceived as threat instead of enrichment. Will my favourite inferences still be part of the ‘definition’ of logic? Will new themes endanger the demiurgical status of the geniuses of the 1930s? Will they make mathematical logic the Aunt, instead of the Queen of the field? Will the new topics dissolve logic into a vast incoherent array of different topics?<sup>25</sup> My view is that there remains one logic, but not in any particular definition of logical consequence, or any favoured logical system. The unity of logic, like that of other creative disciplines, resides in the mentality of its practitioners, and their modus operandi.

Of course, the new psychologism does move the steering wheel a bit as compared with the foundational turn initiated by Frege. Ordinary human reasoning becomes the general topic, with mathematical proof a special case: an important one, to be sure, but still a special case, missing many of the more intriguing features of actual reasoning. Now I do not deny that there is an other-worldly beauty to ‘fundamentalism’, the view that logic should be concerned only with foundations of the sciences.<sup>26</sup> But still, I would pose another conception, which I find more engaging. Frege was obsessed with

<sup>25</sup>I find it hard to think of any academic field which is as defensive as logic. And frankly, it is not a good sign. Fields that are secure in their self-confidence have much less trouble with agenda changes.

<sup>26</sup>Alla Frolova once asked how anyone would want to be a non-classical logician, if one could be a classical one — just as anyone prefers being a Beethoven over a Schoenberg.

foundations, and logic’s providing security, once and for all, for scientific reasoning. To me, however, the key issue is not the static notion of *correctness*, but the dynamic one of *correction*. The most admirable and crucial feature of human intelligence that I see around me is not any ability to be right all the time, but an amazing competence in getting things on the right track once our beliefs, plans, or actions, have gone astray. As Joerg Siekmann once said, the most admirable moments in a mathematics seminar are not when someone presents a well-oiled proof, but when he discovers a mistake and recovers on the spot. Logic should understand this dynamic behaviour, which surely involves many more mechanisms than inference, as discussed in the above. And on that view, logic is not the static guardian of correctness, as we still find it defined in most textbooks, but rather the much more dynamic, and much more inspiring *immune system of the mind!*

I view this as a broader agenda for logic, which leaves everyone their dignity — witness also the passage in Section 4 on different roles concerning the study of logical systems. But I do want to insist that it is not a mere matter of ‘pure’ versus ‘applied’ logic. I find the latter, increasingly popular, terminology very insidious. It fixes by definition what is supposed to be pure logic (usually, the foundational stance in the field), and then declares everything else to be ‘applied’. In that way, the core agenda of logic is fixed forever. In my view, studying information update and belief revision is just as ‘core’ as studying mathematical proof, even though — praise be to our demiurges — it is *their* mathematical techniques developed over the years that still turn out crucial to studying the new agenda.

Finally, here is one more striking fact about the match between logic and reality. There is another way in which the relation between logical theory and human reasoning is not captured well by ‘divergence hunting’. Logical theory provides models for human reasoning, but they are idealized, and may even have creative divergences. But instead of pointing at ‘divergences’, the latter may be important, precisely because they suggest *new practices*, witness computational areas like model checking or automated theorem proving. And again, empirical reality has some nice surprises in store for us here. For, one amazing fact about human cognition are the many ways in which we manage to integrate formally designed practices into our human behaviour. This ‘*insertion*’ of designed practices into our common sense behaviour happens all around us all the time. Think of examples like puzzles or *games*, which can become a reality which feels natural to us — or more technologically —, of a medium like *email*, which enhances our com-

municative capabilities and feels totally ‘natural’ now.<sup>27</sup> Some divergence from the *prima facie* facts around us creates new behaviour that ‘works’, and that creative role of logic, too, is a cognitive reality that should inform the New Psychologism of this *Studia Logica* issue.

## 6. Conclusion

For those who have eyes to see, logic and the empirical cognitive sciences interface today in many interesting ways, and that to mutual benefit. In other words, a ‘Barrier Thesis’ like Frege’s Anti-Psychologism may have worked for a while in keeping the faithful together and at a safe distance from other communities — but reality always seeps through the cracks.<sup>28</sup> Moreover, since abstract theory influences actual behaviour, not just by being right about the cognitive status quo, but also through the design of new intelligent practices allowing for ‘successful insertions’ of behaviour into human lives, the interface between logic and human cognitive practice is much more diverse than the usual normative/descriptive distinction would ever allow us to see. And thus, logic can be so much more than what our Fathers have already made it to be!

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<sup>27</sup> And when all is said and done, think of *mathematics* itself as an example of theory becoming practice: a formal practice with theoretically designed tools, which has come to be felt as natural.

<sup>28</sup> The path of progress in cognitive science is littered with leaking Barrier Theses. Russell’s Misleading Form Thesis is a famous case which failed to keep logic and linguistics apart eventually — but so are famous claims about the purported Inadequacy of Neural Nets (Minsky), or the much-heralded demise of the ‘Symbolic Paradigm’ in the 1980s.

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