# XKCD's NP-Complete

I was reading XKCD along with a friend and we came across a comic.[i] He wanted to continue reading but I wanted to solve it! To start with let's create a general solution:

General solution.

So, we know that the price of the order is 15.05, we know that it consists of an unknown amount of six different appetizers at different prices. I'll also assume that one can't buy a half appetizer or a negative one. In other words: Only positive integers and 0 works. This set is called natural number (denoted by N) but there seems to be some confusion as to whether or not to include 0 in the set of natural numbers.[ii] I'll use N to mean positive integers (denoted $Z_+$) and 0. Symbolized:

Additional assumption.

And so I thought I was nearly done. Wrong. I couldn't figure out how to solve the equation in either MathCad or using my calculator (TI-89 Titanium). Specifically I don't know how to add the requirement that a variable need to belong to a certain set (in this case natural numbers).

But that doesn't mean that it can't be done. I can still do it manually though it would be stupid just trying to brute force guess it. (I.e. trying out all the possibilities.)

First I can calculate the maximum number of an appetizer by dividing the total price with the price of a single appetizer and then removing the digits behind the dot. I'm sure there is a mathematical term for this but I don't recall its name. Symbolically:

Maximum number of a's

Yay! I've found *a* solution. Seven a's will do.

Next question: How many solutions are there? First I'm going to continue the project that I set out to do just before:

Maximum number of b's

Maximum number of c's

Maximum number of d's

Maximum amount of e's

Maximum amount of f's

So, assuming that these are independent, which they are not, we can calculate the number of possible solutions:

Note that all these numbers are one larger than the maximum amount of x's. This is because 0 is also a possibility. A possible solution will look something like this, using the solution found earlier:

Solution 1:

The amount of solutions is not 3840 though (since there are some false ones in the possible solutions set) but it is not larger than that. There are no other simple solutions; I suppose we can call them one-variable solutions since only one variable was not zero. We know there are no other one-variable solutions since it would have showed up when I calculated the maximum amount of x's. A one-variable solution will give an integer, as the first calculation did.

One way to find all the (correct) solutions is to make a computer program that will brute force try all the 3840 possible ones from before. However I can't do this since I can't program that much. I'll just note that it would be relatively easy to do. (Any programmers reading this?)

There is another trick that can be used to narrow the set of possible solutions down. One can look at the digits after the dot. In any correct solution they will be 05. Now, one can look for easy ways to add appetizers together to hit 05. I note is that it will never happen with only any even number of e's or f's or a combination thereof since the last digit in both of them are 0. Also the amount of appetizers with a .05 price is odd, (7 in the found solution.) since if it isn't the sum will not end with a 5 digit, and it has to.

Three c's will make the post-dot digits correct; 3*3.35=10.05. Now this solution "just needs" to add five and there is a correct solution.

I'll leave the project here. Perhaps someone will pick up where I have left. Or perhaps I'll come back later with some better ideas.

i http://xkcd.com/287/

ii http://en.wikipedia.org/wiki/Natural_numbers